Multi-physics Simulation of the Biomedical Process- Heart Electrophysiology

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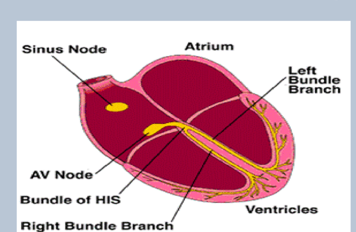
Introduction

- The heart beats from the interaction between electrical, psychological, mechanical, and fluid processes.
- This project studied the electrical process of the heart.
- This project is the stepping stone for other models that could be in the medical field for drug testing pertinent to cardiac failures such as cardiac arrhythmia.



http://heartpoint.com

Purpose

Utilize a set of computer programs to study and simulate the multi-physics phenomena of the heart.

Steps

- Identify the governing equations for simulating the heart: Monodomain Model and Beeler-Reuter Model.
- Develop the geometry and mesh of the heart.
- Program the electrical models: Beeler-Reuter and Monodomain model.
- Examine the interaction between the electrical and physiological effects of the heart.

Formulations and results

- **Monodomain Equations (Tissue)**

$$\frac{\Lambda}{1+\Lambda}\nabla\cdot(M_i\nabla v)=\frac{\partial v}{\partial t}+I_{ion}(v,s)$$

$$\frac{\partial v}{\partial t}=-I_{ion}(v,s),\qquad v(t_n)=v^n$$

$$\frac{\partial s}{\partial t}=f(v,s),\qquad s(t_n)=s^n$$

- **Beeler Reuter Model (Cellular)**

Potential and Current Equations

$$V^{n+1}=V^n+\frac{dV}{dt}\Delta t+O(\Delta t)^2$$

$$\frac{dV}{dt}=-\frac{(i_{k1}+i_{x1}+iNa+iCa-i_{external})}{Cm}$$

$$i_{k1}=\left\{\frac{4\{\exp[0.04(V+85)]-1\}}{\exp[0.08(V+53)]+\exp[0.04(V+53)]}+\frac{0.2(V+23)}{1-\exp[-0.04(V+23)]}\right\}$$

$$i_{x1}=x_1\cdot 0.8\cdot\frac{\{\exp[0.04(V+77)]-1\}}{\exp[0.04(V+35)]}$$

$$iNa=(gNa\cdot m^3\cdot h\cdot j+gNaCa)(V-ENa)$$

Gates Equations (ODE)

$$\frac{dx_1}{dt}=\alpha_{x1}(1-x_1)-\beta_{x1}x_1$$

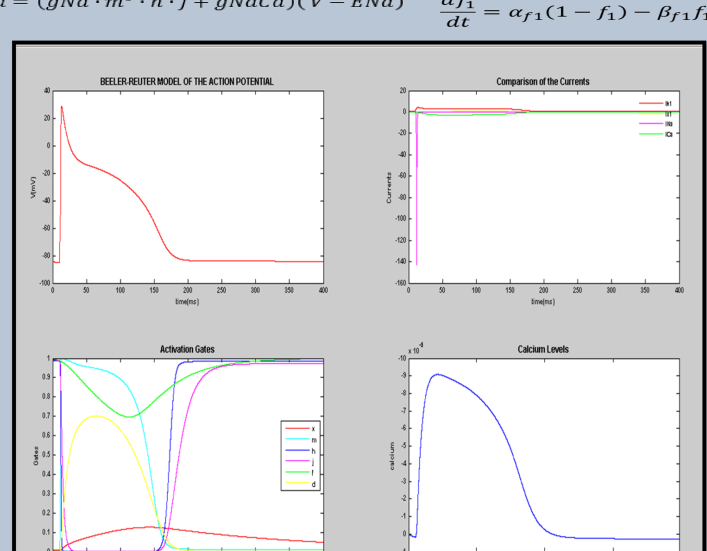
$$\frac{dm_1}{dt}=\alpha_{m1}(1-m_1)-\beta_{m1}m_1$$

$$\frac{dh_1}{dt}=\alpha_{h1}(1-h_1)-\beta_{h1}h_1$$

$$\frac{dh_1}{dt}=\alpha_{h1}(1-h_1)-\beta_{h1}h_1$$

$$\frac{dd_1}{dt}=\alpha_{d1}(1-d_1)-\beta_{d1}d_1$$

$$\frac{df_1}{dt}=\alpha_{f1}(1-f_1)-\beta_{f1}f_1$$

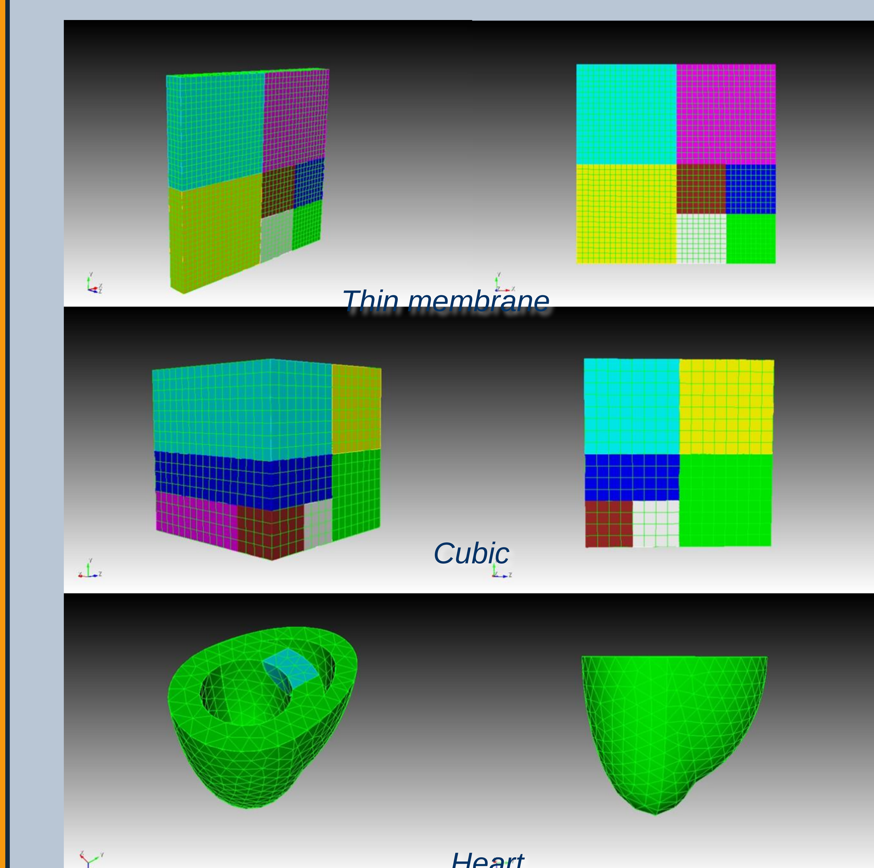


Graph of Beeler-Reuter Model Programmed in Matlab

Meshing

- CUBIT is a full-featured software toolkit for robust generation of two- and three-dimensional finite element meshes (grids) and geometry preparation.
- Three models were developed for this project.

Model	Dimensions(cm)	Node interval (cm)	Mesh Type	No of Nodes	No of elements
Thin membrane	10x10x1	0.281	Hexahedral	7220	5476
Cubic	10x10x10	0.281	Hexahedral	5202	4352
Heart	-	6.51	Tetrahedral	1047	4356



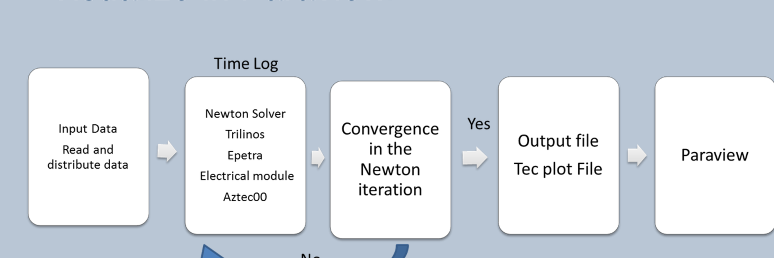
Models developed in Cubit

Programming

- The program is written in C++. The programs runs in parallel.
- Program uses Trilinos and Metis.
- Trilinos is a collection of open source software libraries.
- Libraries in Trilinos used for this program are:
  - Aztec00 for the sparse matrix solver.
  - Epetra for distributing the matrices and vectors to the parallel solver.
- METIS is used for partitioning.

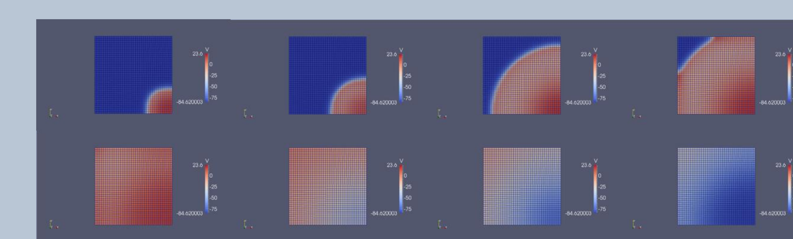
Procedure

- Export the model from Cubit as a Patran file.
- Upload file into program on supercomputer.
- Set parameters.
  - Diffusion
  - Time step
  - Number of processes
  - Runtime
  - Beat length Nodes per element
- Run program.
- Output as Tec plot file.
- Visualize in Paraview.

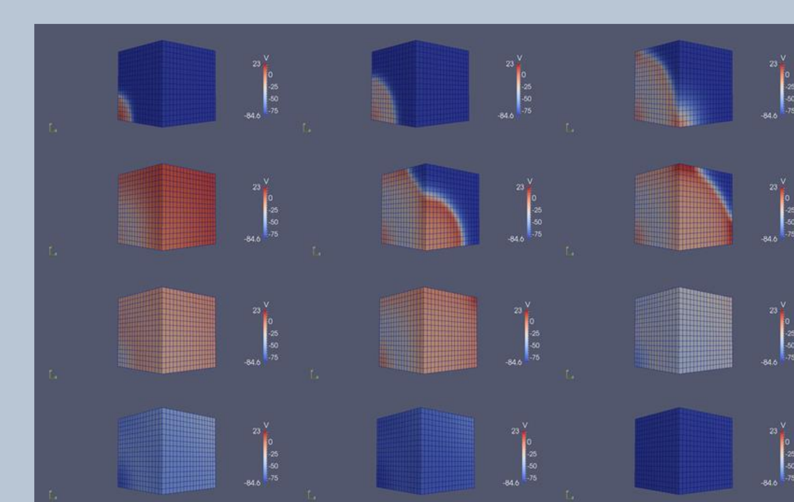


Cases

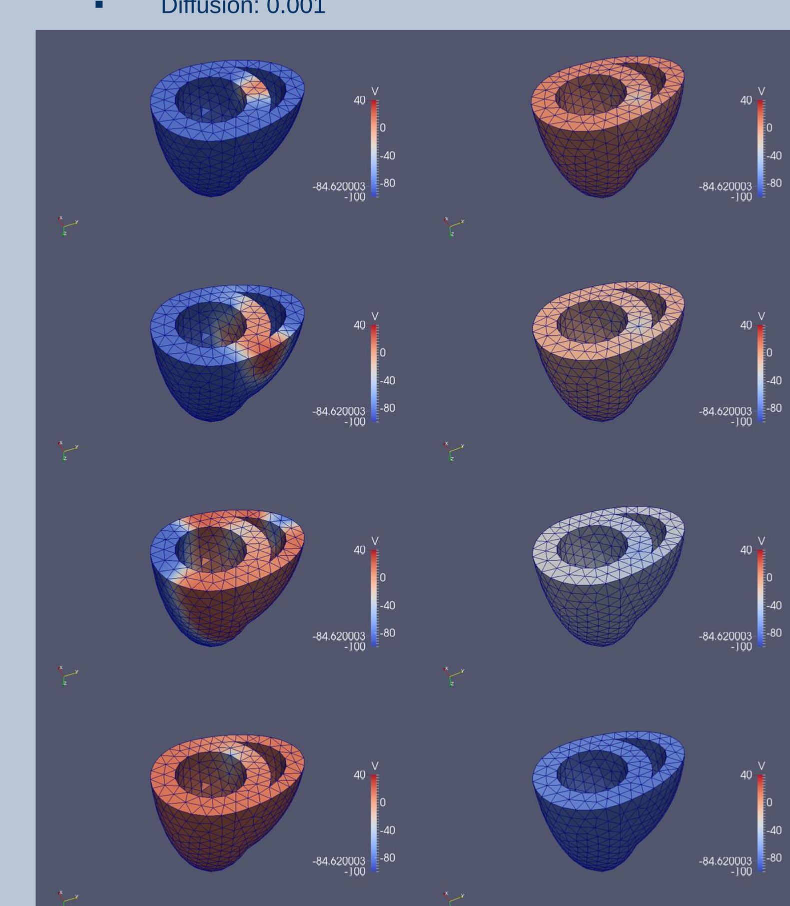
- 1 Beat
  - The models where run to simulate 1 heart beat.
  - The parameters input into the program differed slightly for all models.
  - Thin Membrane
    - Diffusion value:0.01
    - Runtime: 400ms
    - Time step: 1ms



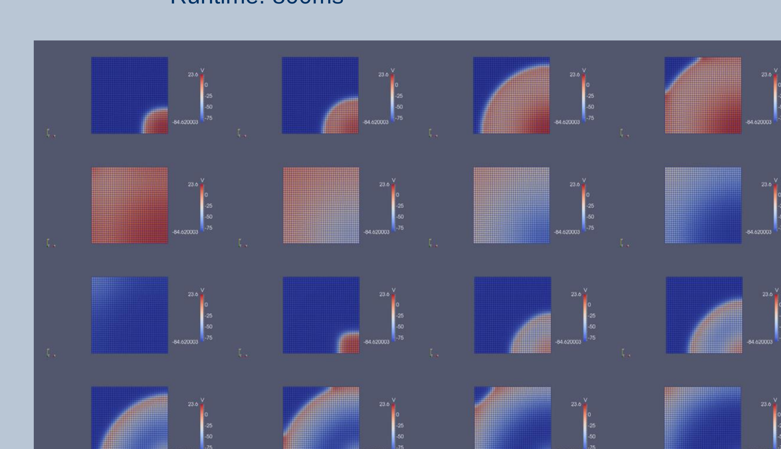
  - Cubic
    - Diffusion value: 0.1



  - Heart
    - Diffusion: 0.001



- 2 Beats
  - The models were run to simulate 2 beats.
  - Only the thin membrane model was able to simulate 2 beats initially
  - Thin Membrane
    - Diffusion: 0.01
    - Runtime: 800ms

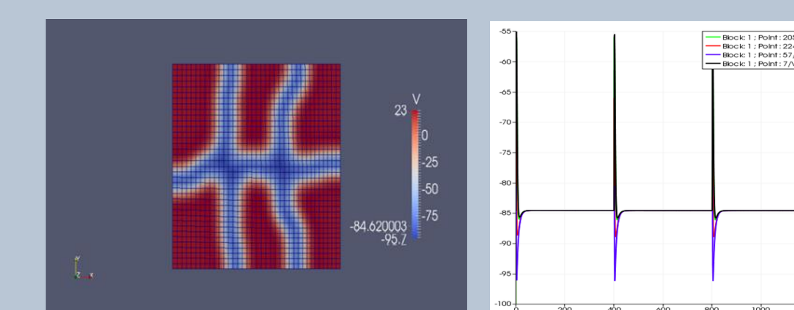


Further Test

- These test were conducted to investigate the why the second beat failed to propagate.

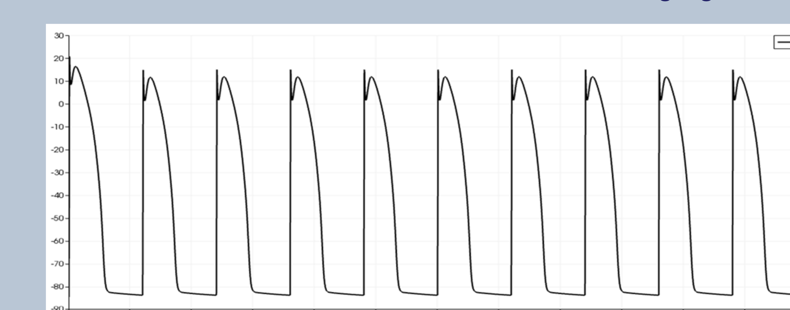
Entire Nodes Simulated Test

- This test was conducted to confirm that the code was either working right or not.
- The plot expected was to be similar to that of the Beeler-Reuter model but did not.



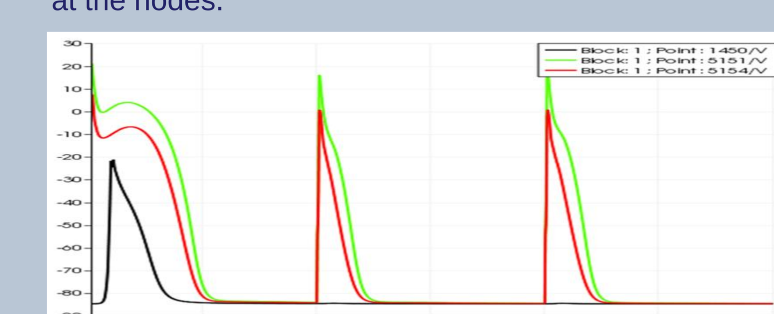
ODE Model Test

- A plot of ten beats was made using only the ODE model.
- This was to test if the ODE model was working right.

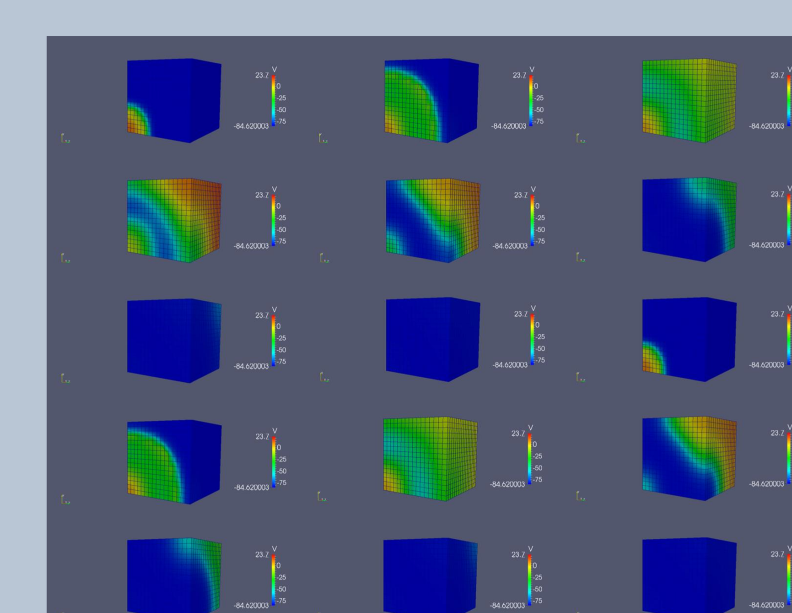


Plot of Nodes

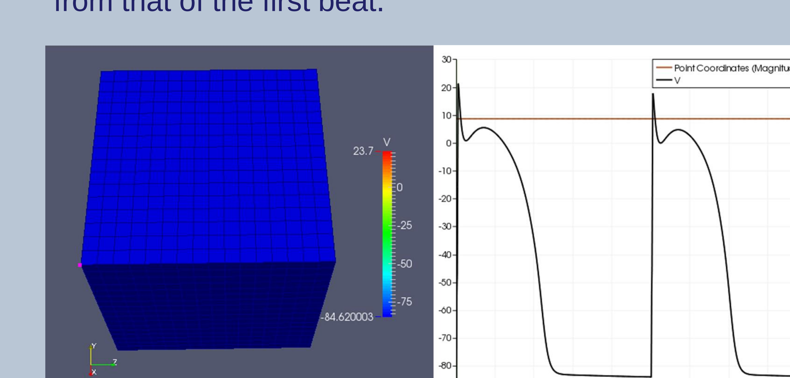
- A plot of nodes in the simulated region, in the middle and at the opposite end of the simulated region was produced.
- The plot showed variations in the second and third beat and at the nodes.



2 Beat Simulation on Cubic Model



- A plot of the node in the stimulated region produced the graph below.
- The second beat did not stimulate as high as the first beat because the initial voltage after the second beat is different from that of the first beat.



Discussions

- The ODE and PDE model work for the first beat in all the cases.
- The ODE and PDE model did not work for the second beat in the cubic and heart cases.
- After running several tests the following conclusions were reached,
  - The coupling between the first and second beat does not occur successfully.
  - The ODE model is very sensitive to the value of the vector potential returning from the first beat.
  - For the second beat to work the ODE model needs to be reinitialized.

Future Work

- Work to correct the current program to simulate multiple beats.
- Incorporate other electrical models to compare the result with the existing model.
- Build upon the current model to adapt the program to simulate cases of Arrhythmia.
- Examine the interactions between the electrical and physiological effects of the heart.
- Improve the geometry and mesh of the current heart model to better represent the heart.

References

- *Sundnes, Joakim, Glenn T. Lines, Xing Cai, Bjorn F. Nielsen, Kent-Andre Mardal, and Aslak Tveito. Computing the Electrical Activity in the Heart. Berlin: Springer, 2006. Print. Ser. 1.*
- *Fenton, Flavio H., Elizabeth M. Cherry, Harold M. Hastings, and Steven J. Evans. "Multiple Mechanisms of Spiral Wave Breakup in a Model of Cardiac Electrical Activity." Chaos: An Interdisciplinary Journal of Nonlinear Science 12.3 (2002): 852. Print.*
- *https://cubit.sandia.gov/*

Acknowledgements

The project is part of the activities in the JICS CSURE REU program sponsored by the National Science Foundation

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